Liquid Propellant Gauging in Low Gravity: the Pressure-Volume-Temperature (PVT) Method

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Fluids in low gravity are controlled by surface tension. Since cryogenic propellants in spacecraft are not held at the bottom of fuel tanks by the Earth's gravity, a reliable and accurate gauging method is essential to mission success. Desirable gauging methods are accurate without requiring settling of the fuel tanks. This reduces fuel margins, reducing both the mass and cost of the mission. The Pressure-Volume-Temperature (PVT) method does not use fuel in the gauging process, works with various tank geometries and liquid placement within the tank, and minimizes the amount of additional hardware required. The PVT method uses a noncondensible gas to pressurize a propellant tank. The mass that is transferred from the pressurant gas supply bottle to the propellant tank is determined to find the percentage of liquid volume that is left within the tank. Analytical and experimental work has been performed using helium as the pressurant gas and liquid oxygen (LO₂) as the propellant. Tests were performed at propellant tank pressures of 50 psia, 150 psia, and 250 psia to verify the accuracy of the PVT gauging method. The data show accuracies within $\pm 3\%$ of full scale or better, thereby demonstrating PVT as a viable gauging method for cryogenic propellants in low-g.

Nomenclature

a1 = compressibility equation temperature coefficient a2 = compressibility equation temperature exponent

AL = applied load, bottom load cell corrected for systematic bias plus the mass of pressurant gas calculated

from the ideal gas law using P_t , T_t , and V_t

 Δm = mass of helium gas transferred to the propellant tank

 m_b = mass of helium gas in the supply bottle

 M_{He} = molar mass of helium

 $m_{i,b}$ = mass of helium gas in the supply bottle, initial condition

 m_L = mass of liquid propellant M_{O2} = molar mass of oxygen

 $\begin{array}{lll} m_T & = & total \ mass \ in \ the \ propellant \ tank \\ m_v & = & mass \ of \ the \ saturated \ vapor \ propellant \\ P_b & = & pressure \ of \ helium \ gas \ in \ the \ supply \ bottle \\ P_{He,t} & = & pressure \ of \ helium \ gas \ in \ the \ propellant \ tank \end{array}$

 $P_{i,b}$ = pressure of helium gas in the supply bottle, initial condition

 P_{ref} = reference pressure

 $\begin{array}{lll} P_t & = & pressure \ of \ the \ propellant \ tank \ ullage \\ P_v & = & saturated \ vapor \ pressure \ of \ the \ propellant \\ \rho_b & = & density \ of \ helium \ gas \ in \ the \ supply \ bottle \\ \rho_{He,t} & = & density \ of \ helium \ gas \ in \ the \ propellant \ tank \end{array}$

 $\rho_{i,b}$ = density of helium gas in the supply bottle, initial condition

R = universal gas constant $<math>R_{He} = helium gas constant$

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SL = sensed load, sum of 3 upper load cells - W_t T_b = temperature of helium gas in the supply bottle

 T_{fill} = average fill line temperature

 T_L = temperature of the propellant tank liquid

 T_{ref} = reference temperature

 $\begin{array}{lll} T_t & = & temperature \ of \ the \ propellant \ tank \\ T_u & = & temperature \ of \ the \ propellant \ tank \ ullage \end{array}$

 T_{vent} = average vent line temperature V_b = volume of the supply bottle

 V_L = volume of propellant in the propellant tank

 V_t = volume of the propellant tank V_u = volume of the propellant tank ullage W_f = fluid weight predicted by correlation W_t = empty propellant tank weight

Z = compressibility factor

I. Introduction

Lautomatically like they are on the ground in Earth's gravity. Fluids in low-g are controlled by surface tension which causes capillary action. When surface tension forces dominate, fluids aren't constrained to expected 1g configurations or locations within containers. For space applications there needs to be accurate and viable methods to gauge cryogenic propellants in propellant tanks without dependence on gravity or settling to locate the liquid at the bottom of the tank.

Space Shuttle methods to gauge cryogenic propellants include firing the engines to accelerate the spacecraft and settle the fuel at the bottom of the tank, enabling the use of level sensors to detect liquid levels, or burn-time integration which is used to determine the volume of propellant left in tanks by integrating a flow meter reading. Both of these methods have significant disadvantages. Settling the tanks burns fuel in the process, adding to the mass of propellant needed for each mission. Integrating a flow meter is an approximate gauging method because the flow meter error accumulates with integration time. The current methods used to gauge cryogenic propellants aboard spacecraft add unnecessary margins and additional cost to the mission. Therefore a new method is desired that would be accurate while not using any propellant in the gauging process.

II. Gauging Methods

There are many gauging methods that are being researched for use with cryogenic propellants in spacecraft, including Optical mass gauge (OMG), radio frequency (RF), and pressure-volume-temperature (PVT)9. Optical mass gauging uses a light source and detector to determine the volume of propellant left in the tank. The liquid absorbs the emitted light but the vapor does not so the light intensity sensed by the detector is inversely proportional to the amount of liquid left in the tank. Advantages of optical mass gauging are that it is fast and no fuel is used during the process. A disadvantage of this method is that hardware in the tank could affect the amount of light sensed by the detector. Radio waves can also be used to sense liquid levels in propellant tanks in low gravity. Different amounts of fluid in the tank produce different resonance responses which can be tested, recorded and used to create a database that can be used for gauging propellants in spacecraft9. Advantages are no settling is required and the method can be virtually instantaneous. However a disadvantage is the response could be affected by the hardware in the tank. Pressure-volume-temperature is a method that uses a noncondensible gas to pressurize the propellant test tank. Pressures and temperatures measured in the helium supply bottle and propellant tank are used to calculate the volume of propellant left in the tank. Advantages of the PVT method are no settling is required, and the method is independent of tank geometry, fluid location, and additional tank hardware. Also, the noncondensible gas and supply bottle are generally a part of spacecraft design. A disadvantage is this method is not instantaneous; time is required for the tank and bottle conditions to become isothermal.

The PVT method is not currently used for applications of cryogenic propellants. It is used with storable propellants for orbiting satellites. Proof-of-concept testing has been performed $^{7.8}$, showing that PVT is a promising method for gauging cryogenic propellants in low-g. Future applications include chemical propulsion technology implemented for the new launch vehicles, Ares I and Areas $V^{2.9}$. Since humans will be traveling back to the Moon

and on to Mars, reliable and accurate methods to gauge propellant levels in propellant tanks are vital for mission success.

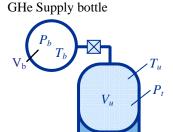
III. Pressure – Volume – Temperature Gauging Method

The pressure-volume-temperature method does not require settling the propellant in order to gauge the volume of liquid left in the tank, which reduces the mass of fuel needed and decreases mission cost. PVT method works in low gravity environments with various tank geometries, and current fuel tanks can be used without a significant increase

in hardware. Testing and analysis was performed to determine the accuracy of this gauging method when applied to cryogenic propellants. The objective is \pm 3% uncertainty of full scale percent fill level or better, when compared to a reference gauging method. The scope of the work reported here includes experimental design analysis, testing, and data analysis.

A. Analytical Method

The Pressure-Volume-Temperature method uses a noncondensible gas to pressurize a propellant tank. The mass of the pressurant gas transferred from a supply bottle to a propellant tank is used to determine the volume of propellant left in the tank. Helium is noncondensible in methane, oxygen, nitrogen and hydrogen due to its lower boiling point. Therefore a helium mass balance is applied to the system of Fig. 1, as shown in Eq. (1), where $\rho_{i,b}$ is the initial density of helium in the supply bottle, V_b is the volume of the helium supply bottle, $\rho_{i,He,t}$ is the initial density of helium in the propellant tank, V_u is the volume of the propellant tank ullage, ρ_b is the density of helium in the supply bottle at a later time, and $\rho_{He,t}$ is the density of helium in the propellant tank.



Propellant tank

Figure 1. Supply bottle and propellant tank. The supply bottle contains a finite amount of helium gas. A valve connects this bottle with a propellant tank containing liquid propellant. Pressures and temperatures are measured in both the supply bottle and propellant tank in order to determine the volume of liquid left in the tank.

$$\rho_{i,b}V_b + \rho_{i,He,t}V_u = \rho_bV_b + \rho_{He,t}V_u \tag{1}$$

Equation (1) assumes solubility of helium gas in the liquid propellant is negligible. Also, since there is no helium initially in the propellant tank, Eq. (1) reduces to Eq. (1a).

$$(\rho_{i,b} - \rho_b)V_b = \rho_{He,t}V_u = \Delta m \tag{1a}$$

By defining Δm as the difference between the initial mass of helium in the supply bottle and the mass of helium in the supply bottle at a later time, the ullage volume, V_u , of the pressurized propellant tank is given by Eq. (1b).

$$V_{u} = \frac{\Delta m}{\rho_{He,t}} \tag{1b}$$

The volume of liquid in the propellant tank, V_L , is the difference between the total tank volume and the volume of the ullage. Then the percentage of liquid filling the propellant tank is given by Eq. (2).

$$\% fill = \frac{V_L}{V_t} \times 100 = \left(1 - \frac{\Delta m}{\rho_{He,t} V_t}\right) \times 100$$
 (2)

In order to find the density of helium in the supply bottle, $\rho_{i,b}$ or ρ_b , the pressure, P_b , and temperature, T_b , are measured and recorded. An equation of state is used to calculate the density of helium as a function of temperature and pressure using the ideal gas law with a compressibility factor, Eq. (3).

$$\rho_b = \frac{P_b}{ZR_{He}T_b} \tag{3}$$

The compressibility factor, Z, is a function of temperature and pressure, Eq. (4).

$$Z = 1 + (a1 * T^{a2})P \tag{4}$$

For the preceding equation temperature is measured in kelvin and pressure is measured in megapascals. The coefficients a1 and a2 are 1.491 and -1.011, respectively.

The temperature and pressure of the tank ullage are also measured and recorded. Since the tank ullage is a mixture of gaseous helium and saturated propellant vapor, the pressure of helium in the tank can be found using Dalton's Law $^{\rm l}$. The pressure of helium in the test tank is the difference between the measured propellant ullage pressure and the partial pressure of the saturated liquid propellant vapor. The saturated vapor pressure of the liquid propellant is a function of the propellant's critical pressure, P_c , critical temperature, T_c , and temperature, T.

Table 1. Coefficients for equation to determine the vapor pressure of oxygen.

k	N_k	i_k	
1	-6.043938	2	
2	1.175627	3	
3	-0.994086	6	
4	-3.456781	14	
5	3.361499	18	

To calculate the saturation pressure of the propellant, the temperature, T, is defined as the temperature of the propellant tank ullage. The vapor pressure can be calculated using Eqs. (5)-(7) and the coefficients listed in Table 1.⁵

$$P_{v} = P_{c} * \exp\left[\frac{T_{c}}{T}f(T)\right]$$
 (5)

$$f(T) = \sum_{k=1}^{5} N_k \theta^{i_k} \tag{6}$$

$$\theta = \left(1 - \frac{T}{T_c}\right)^{\frac{1}{2}} \tag{7}$$

Once the vapor pressure is found, it can be subtracted from the total pressure in the tank to obtain the helium pressure. Then the density of helium in the propellant tank, $\rho_{He,t}$, can be obtained using the equation of state. Tank conditions are assumed to be isothermal.

In order to determine the accuracy of the PVT method, it is necessary to compare these calculations with a reference gauging method. In this case, the reference gauging method only works in 1g; the propellant tank hardware is suspended at the top of the tank by three load cells. The load cell located underneath the test tank was used to measure the applied load during calibration and was assumed to have a zero reading at 2.5 lb. The applied load is defined as the reading from the bottom load cell corrected for systematic bias plus the mass of pressurant gas added to the tank, calculated from the ideal gas law using P_t , T_t , and V_t .

Data was acquired to correlate the weight of fluid in the tank as a function of the sensed load in pounds (the sum of the three load cells minus the empty tank weight), the tank pressure in pounds per square inch, the average fill line temperature and the average vent line temperature, in degrees Rankine. The weight of the empty propellant tank at ambient temperature in vacuum, 1682.5 lb, was determined by subtracting the weight of the pressurant gas at 530°R from the sum of the three load cells suspending the tank. A regression was performed with test data at tank pressures of 50 psia and 250 psia. Each tank pressure was tested with the pressurant at ambient temperature and at a

temperature near cryogenic liquid propellant temperature. Minimum temperatures for the fill and vents lines were 231°R and 287°R, respectively. Each of the four calibration conditions was repeated for a total of eight sets of data used for the regression. A correlation for the fluid weight, W_f , as a function of the sensed load, SL, tank pressure, P_t , the average fill line temperature, T_{fill} , and the average vent line temperature, T_{vent} , is shown in Eq. (8).

$$W_f = A + B * SL + C * (P_t - P_{ref}) + D * (T_{ref} - T_{fill}) + E * (T_{ref} - T_{vent})$$
(8)

The reference temperature was $530^{\circ}R$ and the reference pressure was 14.7 psia. Coefficients A-E obtained from the regression are -6.737, 1.006, -0.06899, -0.002798, and 0.04511, respectively. All 406 predicted values agree with the applied load, AL, to within \pm 8.5 lbs. This is \pm 0.21% of an approximate full scale LO₂ load of 4000 lbs.

A mass balance on the propellant tank is used to determine the fill level of liquid propellant in the propellant tank, shown in Eq. (9) where m_L is the mass of the liquid propellant, m_v is the mass of the saturated vapor propellant, m_{He} is the mass of helium, and m_T is the total mass in the propellant tank.

$$m_L + m_v + m_{He} = m_T \tag{9}$$

Liquid density, ρ_L , liquid volume, V_L , saturated vapor density, ρ_v , helium density, ρ_{He} , and ullage volume, V_u , can be substituted in Eq. (9) to obtain Eq. (9a).

$$\rho_L V_L + (\rho_v + \rho_{He,t}) V_u = m_T \tag{9a}$$

Since the tank volume, V_t , is the sum of the liquid propellant volume, V_L , and the ullage volume, V_u , with further substitution and simplification, Eq. (9b) can be used to solve for the ratio of liquid volume to total tank volume.

$$\frac{V_L}{V_t} = \frac{\left(\frac{m_T}{V_t}\right) - \rho_v - \rho_{He}}{\rho_L - \rho_v - \rho_{He}} \tag{9b}$$

Therefore the ratio of liquid volume to total tank volume is a function of total tank mass, liquid density, vapor density and helium density in the propellant tank. The volume of the propellant tank is 58.2 cubic feet.

Equation (10) can be used to find the liquid density of the propellant, ρ_L , where T_L is the temperature of the liquid propellant, P_t is the pressure of the tank ullage, and a_{T2} , a_{T1} , a_P , and b are the coefficients listed in Table 2. Temperature is measured in kelvin and pressure is measured in megapascals.

$$\rho_L = a_{T2}T_L^2 + a_{T1}T_L + a_P P_t + b \tag{10}$$

The saturated vapor density of the propellant, ρ_v , is a function of the propellant's molar mass, critical density, ρ_c , critical temperature, T_c , and temperature, T_c , and is computed by Eqs. (11)-(13).⁵ For liquid oxygen, coefficients are listed in Table 3.

$$\rho_{v} = \rho_{c} M_{O2} * \exp[f(T)] \tag{11}$$

$$f(T) = \sum_{k=1}^{6} N_k \theta^{i_k}$$
 (12)

$$\theta = \left(1 - \frac{T}{T_c}\right)^{\frac{1}{3}} \tag{13}$$

Table 2. Coefficients for equation to determine the density of liquid oxygen.

LO ₂ density T [K], P [MPa]					
b	1473.7				
a_{P}	2.163				
a_{T1}	-2.374				
a_{T2}	-0.01459				

Table 3. Coefficients for equation to determine the saturated vapor density of oxygen.

k	N_k	i_k	
1	-1.498431	1	
2	-2.116826	2	
3	-0.905713	3	
4	-5.65999	5	
5	-18.90964	12	
6	-53.780774	27	

B. Experimental Method

Testing was performed to verify the accuracy and repeatability of the PVT gauging method. The tank conditions are assumed to be isothermal, which is experimentally achieved by using a cryogenic pump. Tests were performed for three different test tank pressures: 50 psia, 150 psia, and 250 psia. An objective of this work is to compare the method's accuracy over a range of test pressures and perform multiple tests at each pressure to determine repeatability.

C. Test Hardware

The propellant tank (58.2 ft³ when at propellant crvogenic temperatures) shown in Fig. 2 and Fig. 3 is suspended by three load cells and contained within a vacuum chamber to provide insulation and reduce heat transfer into the tank. The bottom load cell is used for measuring the applied load during calibration. Within the propellant tank is a centrifugal pump used to mix the contents of the tank to create isothermal conditions. Silicone diode temperature sensors are located along the central rake, along with the fill and vent lines. Spray bars, located at the top of the tank and lid distribute the fluid circulated by the pump. The diode sensors are located at 5% and 95% of the tank volume, and every 10% between 10% and 90% fill. Clusters of four silicone diode sensors are located at fill levels of 90%, 70%, 50%, 30%, and 10%.

The helium supply bottle (14.4 ft³ volume when cold) is surrounded by

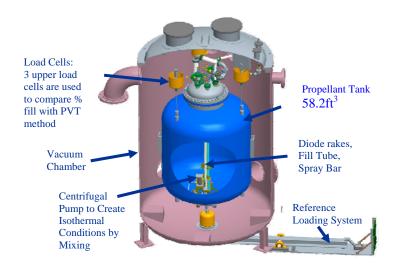


Figure 2. Propellant tank hardware. The propellant tank (58.2 ft^3) is suspended by three load cells and contained within a vacuum chamber. The bottom load cell is used for calibration. Within the tank is a centrifugal pump, silicone diode temperature sensors, and spray bars to distribute the fluid circulated by the pump.

copper coils that circulate liquid nitrogen, cooling the gas for PVT testing. The helium supply bottle and tubing are surrounded by insulation.



Figure 3. Supply bottle (left) and propellant tank (right).

IV. Results

Design analysis tools were created in Excel. For example, given a desired propellant tank pressure, the initial bottle pressure was calculated. Also, assuming a linear temperature profile in a stratified propellant tank, the propellant tank pressure was calculated at 10% fill in the tank. Data analysis tools were created in Excel and Matlab to analyze the raw test data. An Excel tool was created to calculate the % fill for discrete test points at each fill level within the propellant tank using the PVT method. This tool was useful during testing, to determine initial accuracy of the data. A Matlab program was created to calculate the % fill in a propellant tank using the PVT method and the reference load cell method, and calculate the percent error between the two methods. Given a standardized input file of raw test data, it calculates the average supply bottle temperature from three bottle sensors and the density of helium in the supply bottle given the average supply bottle temperature and pressure. It assigns the initial mass of helium in the supply tank as the average of the first twenty values of the mass of the helium in the supply bottle and calculates the change in helium mass. The program then calculates propellant tank conditions such as saturated vapor pressure, saturated vapor density, and liquid density of the propellant. It calculates the pressure of helium in the propellant tank and corrects this value so the minimum pressure has a value of zero, calculates the density of helium in the propellant tank from the same equation of state used in to calculate the density of helium in the supply bottle, and calculates the fluid weight sensed by the upper three load cells. The program exports the input data and calculations, plots the mass of helium in the supply bottle, the change in helium mass, the density of helium in the propellant tank, and the % fill calculated by the load cells. For real time data, the program calculates the ullage volume, uses this to calculate the % fill using the PVT method, and then calculates the error between the PVT method and the load cells.

Figs. 4-9 plot the % fill calculated by PVT and the reference gauging method for two sets of data at each propellant tank pressure: 50 psia, 150 psia and 250 psia. Also listed in Table 4 and plotted in Fig. 10 are results for the error at each fill level between the PVT method and the reference method for each of the six tests.

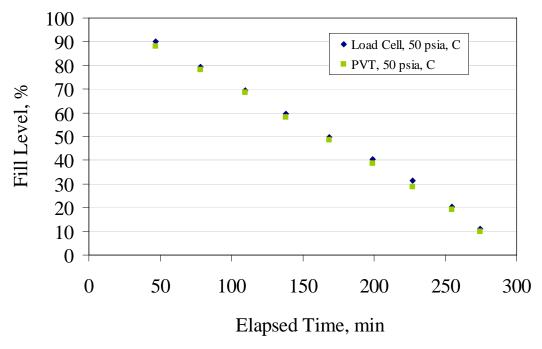


Figure 4. Results of the 50 psia (C) test performed with LO₂.

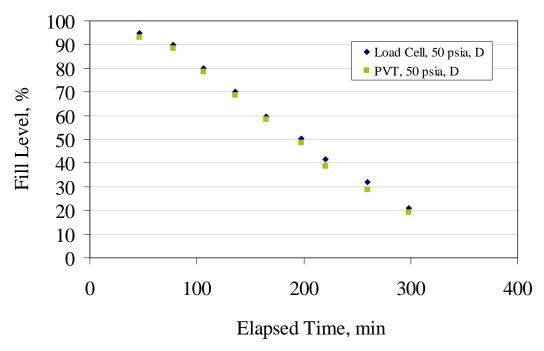


Figure 5. Results of the 50 psia (D) test performed with LO₂.

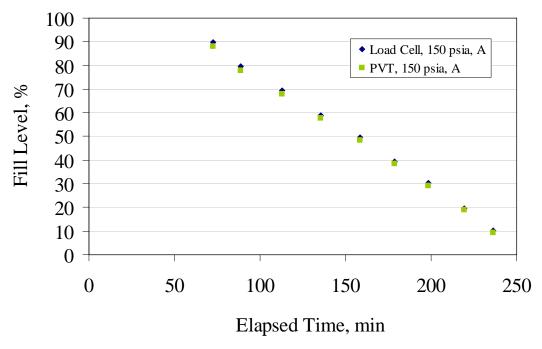


Figure 6. Results of the 150 psia (A) test performed with LO₂.

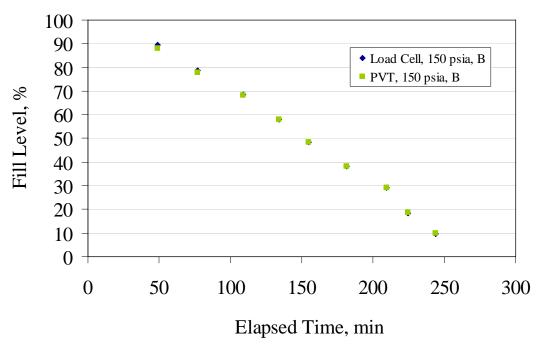


Figure 7. Results of the 150 psia (B) test performed with LO₂.

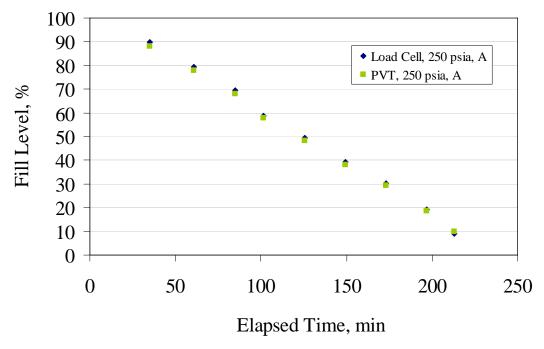


Figure 8. Results of the 250 psia (A) test performed with LO₂.

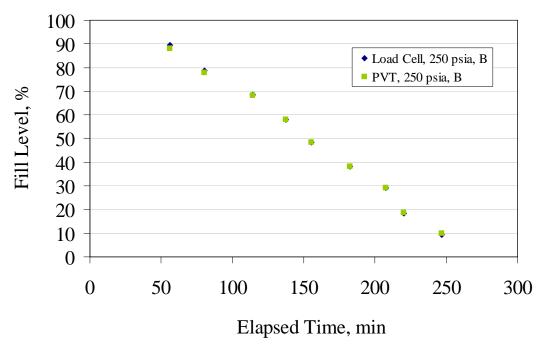


Figure 9. Results of the 250 psia (B) test performed with LO₂.

Table 4. Results of the 50 psia, 150 psia, and 250 psia tests performed with LO₂.

Difference (PVT vs. Load Cells)

	50 psia	50 psia	150 psia	150 psia	250 psia	250 psia
%fill	C	D	A	В	A	В
90	2.0	1.9	1.6	1.4	1.9	1.4
80	1.4	1.6	1.6	0.7	1.6	0.7
70	0.9	1.5	1.4	0.3	1.4	0.3
60	1.5	1.4	1.2	0.2	1.1	0.2
50	1.2	1.2	0.9	0.0	1.0	0.1
40	1.8	1.6	1.1	0.0	1.0	0.0
30	2.6	2.7	1.3	-0.1	1.0	-0.1
20	1.3	3.2	0.7	-0.4	0.6	-0.2
10	1.1	1.7	0.7	-0.3	-0.8	-0.4
min>	0.9	1.2	0.7	-0.4	-0.8	-0.4
ave>	1.5	1.9	1.1	0.2	1.0	0.2
max>	2.6	3.2	1.6	1.4	1.9	1.4

Difference (PVT vs. Load Cells)

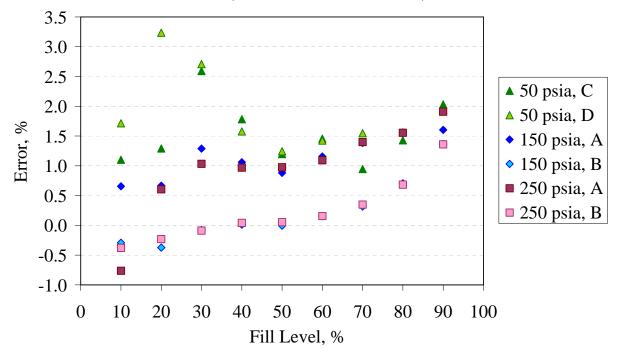


Figure 10. Results of the 50 psia, 150 psia, and 250 psia tests performed with LO₂.

V. Conclusion

The pressure-volume-temperature method is accurate to within the desired \pm 3%. Most of the data is accurate to within \pm 2% or better as compared to a reference gauging method. There is good agreement between trials at the same propellant tank pressures (Table 4). For the 50 psia tests, Figs. 4-5, the PVT calculations consistently predicted lower fill levels than the load cell calculations. Since the PVT method overestimated the ullage volume, this discrepancy could be due to calculating a lower density of helium in the propellant tank. Higher pressures of 150 psia and 250 psia showed more accurate results than testing performed at 50 psia (Fig. 10, Table 4). For all of the tests, the maximum spread between minimum and maximum error was 2.7% (Table 4). One explanation for the error between the PVT method and the load cells may be the solubility of the pressurant gas in the liquid propellant. Solubility would make a difference especially at high fill levels; with more liquid in the tank, more of the pressurant gas is able to go into solution.

Future work focuses on refining the data analysis and determining the actual measurement accuracy. This includes adding the effect of helium solubility in the liquid propellant. The solubility is assumed to be proportional to the time the pump is running and mixing the tank's contents. Another goal is to be able to model stratified temperature conditions within the test tank. The advantage would be that the method would require less time to gauge the propellant. If conditions within the test tank don't need to be isothermal this also means it isn't necessary for the pump to run as often or possibly at all. In a spacecraft, this would save mass, increasing the allowable payload mass, and reduce mission cost. It would be necessary to verify the model and confirm that the accuracy of the PVT is still high enough when the test tank is not isothermal. Secondly, a discrepancy currently exists between the volume sensed by the load cells and that calculated by the PVT method. In practice, the volume of the test tank includes the volume of the fill and vent lines in the tank, which extend above the load cells and to other supports. A goal would be to explain and account for the difference between the PVT and load cell methods. Thirdly, the volume of components connected to but external to the tank are warmer than inside the tank, creating nonisothermal conditions. Fourthly, the load cell calibration should be repeated to verify its accuracy. Additionally, the tests with LO₂ should be compared to previous work using LN₂^{7,8} and presented. Even though nitrogen is not a cryogenic fuel it was tested because its properties are similar to propellants such as oxygen and it has fewer safety concerns. Further work includes plans to test the PVT method with methane, CH₄, and in low-g environments such as on low-g aircraft and in space, before its implementation with cryogenic propellants in spacecraft propellant tanks.

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